Published by Institute of Physics Publishing for SISSA
Received: *July 27, 2005*
Accepted: *August 26, 2005*
Published: *September 1, 2005*

DBI with primordial magnetic field in the sky

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Abstract: In this paper, we study the generation of a large scale magnetic field with amplitude of order μG in an inflationary model which has been introduced in [1]. This inflationary model based on existence of a speed limit for inflaton field. Generating a mass for inflaton at scale above the ϕ_{IR}, breaks the conformal triviality of the Maxwell equation and causes to originate a magnetic field during the inflation. The amplitude strongly depends on the details of reheating stage and also depends on the e-foldings parameter N. We find the amplitude of the primordial magnetic field at decoupling time in this inflationary background using late time behavior of the theory.

Keywords: AdS-CFT and dS-CFT Correspondence, Brane Dynamics in Gauge Theories, D-branes.

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Contents

1. Introduction

The observational data of galaxies clusters, disks and spiral galaxies displays a large scale magnetic field with coherence scale of few kpc and an amplitude of order μG ([2]–[13]). Such amplitudes present an "inverse" fine tuning problem as compared with the standard one in inflation [2].

Although, the origin of such seed field is not exactly known, but several mechanism for originating the magnetic field have been supposed and some limits on its amplitude has been found. Magnetic field can be produced by charge separation mechanism during galaxy formation, or by amplification of preexisting seed field. In the second case, the amplification can be achieved either by the adiabatic compression of the fields in the collapse of the protogalactic cloud, or by the galactic dynamo mechanism where the differential rotation of the galaxy is able to transfer kinetic energy into magnetic field. In the latter case, the limit on the primordial seed fields at the decoupling time is in the range $B_{\rm dec} \sim 10^{-17}-10^{-20}G$ [3] for flat universe without cosmological constant. For a flat universe with cosmological constant the limits are relaxed up to $B_{\rm dec} \sim 10^{-25} - 10^{-30}G$ [4]. Such seed field is amplified exponentially by galactic dynamo, or other mechanism, until it reaches to the observed value $B_{\rm dec} \sim 10^{-6}G$.

However, any homogenous magnetic field existing before decoupling time should be weaker than $B_0 \leq 10^{-9}G$ in order to avoid the production of large anisotropies in the cosmic microwave background radiation [4].

All the above mechanism need a primordial seed field. There are two main explanation for generating such a field. On the one hand, those based on the phase transition in the early universe. The smallness of the field coherence is the main problem of these theories.

On the other hand, the electromagnetic quantum fluctuation during inflation is another source for amplifying the field [5]. The main problem in this case is that in order to get some amplification it is necessary to breaks the conformal symmetry of the Maxwell equation in Friedmann-Robertson-Walker background. In fact, there is no mixing between positive and negative frequency modes of the solution of Maxwell equation, hence there is no photon production [4]. Therefore, in order to get magnetic field amplification we need to break conformal invariance.

There are several possible sources for breaking the conformal invariance for example: non zero masses of charged particles, absence of conformal invariance for even massless scaler field, quantum conformal anomaly due to the triangle diagrams, and breaking the U(1) gauge invariance as well as conformal invariance [7].

Although, the generation of magnetic field in the standard model of cosmology was studied in various frameworks it was also studied in string cosmology [10 − 13]. String cosmology provides a very natural way for breaking the conformal triviality of the Maxwell equation. In fact, the story was started when the author of [14] found a warped geometry in the presence of fluxes and branes. By adding anti$-D3$ brane to these solution, one can lift the ADS vacuum to DS vacuum and find locally stable minimum (KKLT vacua). So, it is very natural to study inflation in string theory by considering the KKLT vacua. Many different efforts was done to finding an inflationary model in string theory in which is in agreement with observational data [1, 15, 17, 18].

Furthermore, one may suppose the fluctuation of gauge field via DBI action in a typical inflationary background and study the dynamic of such field. In inflationary model which we are interested in [1, 15], conformal invariance breaks in some region and magnetic field can be produced in the early universe.

In the following subsection 1.1, we will present a preview of this inflationary model which based on D-cceleration mechanism for inflation, then in section 2 we'll study the equation of motion of the gauge field and study it by using the late time behavior of various parameters and fields in this inflationary background and find the amplitude for the magnetic field in term of e-foldings parameter. Some comments about reheating stage will be presented in section 3.

1.1 Review of D-cceleration

In this model, "D-cceleration" mechanism in strongly coupled conformal field theories causes for slowing motion of scalar field [1, 15]. The dynamic of the field ϕ in strongly 't Hooft coupled conformal field theory is easy understood in gravity side of the AdS/CFT correspondence [19].

In CFT side, the slowdown arising from the virtual effects of the light particles. Moreover, the integrating out of such χ fields produces higher derivative terms in the effective action which are captured in DBI action and due to speed limit on rolling down of scalar field.

In gravity side, a probe D_3 brane travelling down a five-dimensional warped throat and the inflaton field ϕ is a scalar parameterizing a direction on the approximate Colomb branch of the system. Motion toward the origin is motion in the radial direction of the

AdS_5, toward the horizon where new degrees of freedom χ becomes light. The motion of ϕ is constrained by the causal speed limit on the gravity side, leading to slow roll even in the presence of a steep potential.

The dynamic of the probe D_3 brane coupled to gravity is given by DBI action [16, 1],

$$S = \int \frac{1}{2} M_P^2 \sqrt{-g}\Re + \pounds_{\rm eff} + \cdots \,, \tag{1.1}$$

where

$$\pounds_{\rm eff} = -\frac{1}{g_s}\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1 + f(\phi) g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \sqrt{f} g^{\mu\nu}F_{\mu\nu}} + V(\phi)\right). \tag{1.2}$$

Here $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$. The function $f(\phi)$ is the (*squared*) warped factor of the AdS-like throat. For example, for a pure AdS_5 of radius R, it is $f(\phi) = \frac{\lambda}{\phi^4}$ with $\lambda \equiv \frac{R^4}{\alpha'^2}$.

The potential $V(\phi)$ arises from the coupling of the D brane to background RR flux. In the case of $\mathcal{N} = 4$ theory, it is quartic and conformal invariance prohibits the generation of a mass term for ϕ. But, for this approximate AdS throat, conformal invariance holds only at scale above ϕ_{IR} and there is nothing preventing the generation of a mass for ϕ in which

$$V(\phi) = m^2\phi^2 \,. \tag{1.3}$$

This leads to a χ mass of order $\sqrt{g_s}m$ cutting off the throat, so the mass satisfies $m \leq \frac{\phi_{IR}}{\sqrt{g_s}}$. One can also add cosmological constant term to the action (1.1) using KKLT vacua.

Considering the flat FRW metric,

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j \tag{1.4}$$

and using the Hamilton-Jacobi approach in which the field ϕ roles as cosmological time, one obtains [1, 15],

$$\begin{aligned} \dot\phi &= -2g_s M_P^2 \frac{H'(\phi)}{\gamma(\phi)} \,, \\ V(\phi) &= 3g_s M_P^2 H(\phi)^2 - \frac{\gamma(\phi)}{f(\phi)} + \frac{1}{f(\phi)} \,, \end{aligned} \tag{1.5}$$

where

$$\gamma(\phi) = \frac{1}{\sqrt{1 - f(\phi)\dot\phi^2}} = (1 + 4g_s^2 M_P^2 f(\phi) H'(\phi)^2)^{1/2} \,, \tag{1.6}$$

and the slow roll parameter is

$$\epsilon = \frac{2g_s M_P^2}{\gamma}\left(\frac{H'}{H}\right)^2 . \tag{1.7}$$

The late time behavior of the scale factor $a(t)$ is

$$a(t) \to a_0 t^{1/\epsilon} \,. \tag{1.8}$$

We see that for $\epsilon < 1$ we have a power low inflation. A detailed analysis shows that

$$\frac{1}{\epsilon} = \frac{1}{3}\left(1 + \sqrt{1 + \frac{3m^2\lambda}{g_s M_P^2}}\right) \simeq \sqrt{\frac{\lambda}{3g_s}}\frac{m}{M_P} \tag{1.9}$$

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and the condition $\epsilon < 1$ means that $m^2\lambda > g_s M_P^2$. In the other hand, accelerated expansion occurs only if the mass of inflaton m is suitably large compared with Planck mass.

The number of e-foldings parameter is given by

$$N = \frac{1}{\epsilon}\log\left(\frac{\phi_i}{\phi_e}\right), \tag{1.10}$$

where ϕ_i and ϕ_e are the inflaton field at the begin and the end of the inflation respectively. There are possibilities to start and end inflation anywhere within the ADS-like geometry, subject to $\phi_{IR} \leq \phi_{end} < \phi_{start} \leq \phi_{UV}$, where ϕ_{IR} and ϕ_{UV} are determined by the geometry of the throat. The UV end of the throat arises at the scale $\phi_{UV}\alpha' \sim R$ or $\frac{\phi_{UV}}{\sqrt{g_s}} \sim \frac{\lambda^{1/8}M_P\sqrt{g_s}}{\sqrt{V_X}}$, where V_X is the volume in string unit of X. Note also that $\phi_{IR} > \sqrt{g_s}m$.

2. DBI with magnetic fluctuation

The D-3 brane dynamic in the ADS geometry is described at low energy by DBI action (1.2), which is the Yang-Mills action plus some higher derivative corrections. Considering (1.2) and using the expansion

$$\begin{aligned}\sqrt{\det(M_0+M)} = \sqrt{\det(M_0)}\Big[1 &+ \frac{1}{2}\,\mathrm{Tr}(M_0^{-1}M) - \frac{1}{4}\,\mathrm{Tr}(M_0^{-1}M)^2 + \\ &+ \frac{1}{8}(\mathrm{Tr}(M_0^{-1}M))^2 + \frac{1}{6}\,\mathrm{Tr}(M_0^{-1}M)^3 - \\ &- \frac{1}{8}\,\mathrm{Tr}(M_0^{-1}M)\,\mathrm{Tr}(M_0^{-1}M)^2 + \frac{1}{48}(\mathrm{Tr}(M_0^{-1}M))^3 + \cdots\Big],\end{aligned} \tag{2.1}$$

one finds the final action up to second order of gauge field as

$$-\frac{1}{g_{YM}^2}\int d^4x\left[\gamma^{-1}\left(\frac{1}{2a}(\nabla\times\vec{A}).(\nabla\times\vec{A}) - \frac{a}{2}\dot{\vec{A}}^2\gamma^2\right) + f^{-1}(\gamma^{-1}-1) + V(\phi)\right]. \tag{2.2}$$

Here we used the Coulomb gauge ($A_0 = 0 = \vec{\nabla}.\vec{A}$). The variation of the action over the field $\vec{A}$ gives us the following equation of motion

$$\ddot{\vec{A}} + \dot{\vec{A}}\left[H + \frac{\gamma^2}{2}(f'\dot{\phi}^3 + 2f\dot{\phi}\ddot{\phi})\right] - \frac{1}{\gamma^2a^2}\nabla^2\vec{A} = 0\,, \tag{2.3}$$

Using the standard trick, which is briefly explained at the appendix, this differential equation can be written as

$$\tilde{\vec{A}}'' + [k^2 - f(\eta)]\tilde{\vec{A}} = 0\,, \tag{2.4}$$

where $\eta = \int a^{-1}dt$ is the conformal time and new variable $\tilde{\vec{A}_k}$ is defined as $\vec{A}_k \equiv b^{\frac{-1}{2}}\tilde{\vec{A}_k}$ and we have

$$\begin{aligned}f(\eta) &= k^2f\dot{\phi}^2 + \frac{a^2}{4}[(F^2-H^2) + 2(\dot{F}-\dot{H})] \\ F &= H + \frac{\gamma^2}{2}\left(f'\dot{\phi}^3 + 2f\dot{\phi}\ddot{\phi}\right).\end{aligned} \tag{2.5}$$

The solution for equation (2.4) is

$$\tilde{\vec{A}}_k = \alpha_k \tilde{\vec{A}}^i_k + \beta_k \tilde{\vec{A}}^{*i}_k \,, \tag{2.6}$$

where the Bogolyubov coefficient is

$$\beta_k = -i \int_{\eta_i}^{\eta_f} \tilde{\vec{A}}^i_k f(\eta') \tilde{\vec{A}}_k d\eta' \,. \tag{2.7}$$

The quantities $\tilde{\vec{A}}^i_k$ are the initial value for the field $\tilde{\vec{A}}_k$. It is easy to see that at asymptotic past time $\eta \to -\infty$ the $f(\eta)$ term goes to zero and the solution is $\tilde{\vec{A}}^i_k = e^{-ik\eta}/\sqrt{2k}$.

The energy density stored in a magnetic field is given by

$$\rho_B = \frac{|B_k^2|}{8\pi} \,. \tag{2.8}$$

We want to find the amplitude of magnetic field at scale corresponds to decoupling time, $\omega_{\rm dec} = \frac{k_{\rm dec}}{a_{\rm dec}}$. Then

$$|B_k| = \sqrt{8\pi}(\omega_{\rm dec})^2 |\beta_k| \,. \tag{2.9}$$

In this step, one may use the procedure that one applied in [11], by defining a dimensionless quantity α. Similarly, but with small changes, we want to find the amplitude by finding the late time behavior of the $f(\eta)$.

2.1 Evaluation of $f(\eta)$

Let us compute the late time behavior of $f(\eta)$ using (2.5). The equation of motion for ϕ, by neglecting the gauge field effects, is [1]

$$\ddot{\phi} + \frac{3f'}{2f}\dot{\phi}^2 - \frac{f'}{f^2} + \frac{3H}{\gamma^2}\dot{\phi} + \left(V' + \frac{f'}{f^2}\right)\frac{1}{\gamma^3} = 0 \,. \tag{2.10}$$

The late time behavior of scale factor is power law as $a = a_0 t^{1/\epsilon}$. In this case, considering slow roll condition $\gamma \gg 1$, (which means that$f \sim \dot{\phi}^2$) and using (1.5), (1.8), (2.5), (2.10) and the late time behavior of ϕ, γ, H as

$$\phi \to \frac{\sqrt{\lambda}}{t} \,, \qquad \gamma \to \sqrt{\frac{4g_s}{3\lambda}} M_P m t^2 \,, \qquad H \to \frac{1}{\epsilon t} \,, \tag{2.11}$$

one finds following expression for the late time behavior of f in term of conformal time η,

$$f(\eta) \to k^2 + \sum_{\beta=1}^{\beta=5} f_\beta h_\beta \eta^{\frac{-2(1-\beta\epsilon)}{1-\epsilon}} \,, \tag{2.12}$$

where

$$h_\beta = \left(m a_0^{\frac{\epsilon}{\epsilon-1}}\right)^{-2(\beta-1)} \,, \tag{2.13}$$

and f_β's are functions of ϵ as

$$
\begin{aligned}
f_1 &= \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{2(1-\epsilon)}{\epsilon-1}}\left[-\frac{24\epsilon+3}{2\epsilon^2}-\frac{7\epsilon+1}{2\epsilon^2}(9-6\epsilon)^{1/2}\right],\\
f_2 &= \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{2(1-2\epsilon)}{\epsilon-1}}\left[\frac{9\epsilon+1}{2\epsilon^2}(9-6\epsilon)^{1/2}+\frac{9-6\epsilon}{\epsilon^2}\right],\\
f_3 &= \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{2(1-3\epsilon)}{\epsilon-1}}\left[\frac{-8\epsilon^2-6\epsilon+27}{8\epsilon^2}(9-6\epsilon)+\frac{9}{8\epsilon^4}(9-6\epsilon)^{3/2}\right],\\
f_4 &= \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{2(1-4\epsilon)}{\epsilon-1}}\left[-\frac{9}{4\epsilon^4}(9-6\epsilon)^{3/2}-\frac{(9-6\epsilon)^2}{2\epsilon^4}\right],\\
f_5 &= \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{2(1-5\epsilon)}{\epsilon-1}}\left[\frac{(9-6\epsilon)^2}{2\epsilon^4}\right].
\end{aligned}
\tag{2.14}
$$

The constant a_0 which arises in time evaluation of scale factor is a model dependent quantity. In fact, as it was mentioned in [15], the initial value for ϕ at starting point is constrained within the consistence between predicted non-gaussianity of the theory and non-gaussianity data from WMAP analysis. For example, for the case $\epsilon = 1/20$, which is our interested case, the initial value for inflaton field is $\phi_i \sim 1.5\sqrt{g_s}M_P$ [15]. From this, the amount of the a_0 depends on details of the reheating, the value of ϕ at starting time and at the end of the inflation. In the other word, the e-folding parameter has a crucial roll in the a_0. Considering (1.10), (2.13), one obtains following a_0

$$
a_0 = \left(\frac{\pi^2 g_* \epsilon^2 \lambda^{\frac{\epsilon-2}{\epsilon}}}{90 g_s M_P^2 \phi_i^{\frac{2\epsilon-4}{\epsilon}}}\right)^{1/4} a_{\rm dec} T_{\rm dec} e^{(\frac{\epsilon}{2}-1)N}\,, \tag{2.15}
$$

where we supposed that at the end of inflation, the energy of the inflaton field transformed to the gauge field completely $i.e$ we used at the end of the inflation $3g_s M_P^2 H^2 = \frac{\pi^2}{30} g_s T_{Reh}$. We also used the fact that at radiation dominated era $a \sim \frac{1}{T}$.

Here, g_* is the number of degrees of freedom and $T_{\rm dec}$ is the temperature at decoupling time.

Using $\phi_i \sim b\sqrt{g_s}M_P$ one finds

$$
h_\beta = \left(b^{\frac{-2\epsilon+4}{\epsilon}}\frac{\pi^2 g_* \epsilon^2}{90 g_s}\right)^{-\frac{\epsilon(\beta-1)}{2(\epsilon-1)}}\left(\frac{m}{M_P}\right)^{-2(\beta-1)}\left(\frac{\lambda}{g_s}\right)^{-\frac{\beta-1}{2}\frac{\epsilon-2}{\epsilon-1}}(a_{\rm dec}T_{\rm dec})^{-\frac{2(\beta-1)\epsilon}{\epsilon-1}}\, e^{-(\beta-1)\epsilon\frac{\epsilon-2}{\epsilon-1}N}\,. \tag{2.16}
$$

where b is a constant. One can also find expressions for the η_i and η_f using a_0 similarly. Inserting (2.14) and (2.16) in (2.12) one find the final expression for the $f(\eta)$ in terms of e-folding parameter N.

In the next section we will use the observational data for evaluate $f(\eta)$ numerically and find the amplitude of the $B_{\rm dec}$.

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2.2 Estimating the quantities

First of all, at decoupling time $\omega_{\rm dec} = k_{\rm dec}/a_{\rm dec} \simeq 10^{-33}\,{\rm GeV}$, and $T_{\rm dec} \sim 10^{-10}\,{\rm GeV}$.

Although, it would be nice to study the generation of the magnetic field during inflation for generic ϵ, but in this paper, by considering the WMAP analysis [21], we focus on the smallest ϵ consistent with a Plank scale inflaton vev which is $\epsilon \sim 1/20$. For this case we have $b = 3/2$. In this case, the COBE normalization requires a mass hierachy $\frac{m}{M_P} \simeq 2 \times 10^{-5}$ [15]. The quantity λ/g_s corresponds to the number of D-3 brane. For the current case, we have $\frac{\lambda}{g_s} \sim 10^{12}$, which corresponds to an enormous gauge group.

We choose $g_s \sim 1$ because we are at the weakly coupled gravity regime. We also have $g_* \simeq 106.75$ and $\frac{1G^2}{8\pi} = 1.9089 \times 10^{-40}\,{\rm GeV}^4$.

In this inflationary model, the largest Hubble parameter is

$$H \simeq \frac{1}{\sqrt{3g_s}}\frac{m\phi}{M_p} \simeq \frac{1}{\sqrt{3}}\frac{m^2}{M_P}\,. \tag{2.17}$$

This corresponds to maximum available value for e-foldings parameter $N \leq 200$. From the usual requirement of solving the flatness and homogeneity problems, we need at least 60 e-foldings in total, but the 60 e-foldings could comes from multiple passes through the inflationary phase since the scale invariant spectrum of the CMBR accounts for only about 10 e-foldings of observations. So $10 \leq N \leq 200$.

We find the amplitude for the minimum available e-foldings $N = 10$ using numerical program and obtain

$$B_{\rm dec} \simeq 3.7 \times 10^{-18} G\,, \tag{2.18}$$

which is in good agreement with observational data.

3. Reheating stage

The energy density for the scalar field ϕ decrease during the inflation as $\rho \to \frac{3g_s M_P^2}{\epsilon^2 t^2}$. In fact, as the theory of inflation and reheating, the energy of the inflaton field is released to the other field at the end of the inflation and then the reheating stage started.

At reheating stage the frequency modes greater than the horizon size is allowed in which one may study the effects of the modes deep inside the Hubble radius ($k \gg a_e H_e$). The energy stored in this modes surpasses the energy of the inflaton field and the reheating procedure is done.

In D-cceleration model, the end of inflation is not very clear. There are different possibility for ending the inflation. In the other word, when the brane reaches the end of the throat, it can oscillates of the form discussed in [22], or annihilates using another anti D_3 brane or other novel phenomena can be happened.

The back reaction of closing of the brane to other branes and Tachyon production are other effects that one should consider at the end of such inflationary models.

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In the CFT side the reheating stage correspond to the presence of new light particles χ which was integrated out. In fact, one may consider in $\mathcal{N} = 4$ super Yang-Mills theory, ϕ as a Higgs particle breaking the $\mathrm{U}(N)$ gauge group to $\mathrm{U}(N-1)$. Reaching the value of ϕ to zero means that Higgsed gauge boson χ becomes massless and the gauge symmetry restored. So at reheating stage the effects of virtual particles are important.

So, the end of inflation in this model is rather complicated and one should consider all the above phenomena for studying reheating stage.

4. Conclusion

Using ADS/CFT, an inflationary model was introduced in string theory in [1] which based on D-cceleration mechanism. In fact, the author have been noticed the effects of the virtual gauge bosons at the strongly coupled CFT. In gravity side, the problem translated to the motion of the probe D-3 brane toward the horizon. The causal structure of the theory causes a speed limit on rolling down of scalar field and power low evaluation of scale factor at late time. They find for suitably large inflaton mass $m^2\lambda > g_s M_P^2$ there is such power law inflation.

The problem of existence of a large scale magnetic field observed in the galaxies is an unsolved problem for a long time. In fact, the smallness of the field coherence or the conformal triviality of the Maxwell equation are some main difficulties to obtain a good theoretical explanation for this problem. String cosmology provides a natural framework for studying this problem.

In this paper, we study the production of the primordial magnetic seed field during inflation considering the above inflationary model. By coupling the gauge field with scalar field ϕ by DBI action at weakly coupled supergravity, the equation of motion of gauge field was derived. The generation of mass for the inflaton field via the coupling of $3-$brane with background RR fluxes, causes that the conformal invariance of the Maxwell equation breaks. Then, we write the equation in a standard form and study the solution of this equation at late time. We finally find the amplitude for the magnetic field generated during inflation which is in good agreement with observational data.

There is an important comment. The amount of the B_k^{dec} is strongly related to the $\epsilon, \lambda/g_s$ and the mass hierachy m/M_P. For example, for the case $\epsilon \simeq 1/20$, considering the non-gaussianity with the WMAP analysis, the starting point for the ϕ field is slightly above the Planck scale ($\phi \sim 1.5\sqrt{g_s}M_P$). In this case it would entail a functional fine tuning of the parameters as in early models of ordinary inflation[15]. This can be avoided by studying the generation of magnetic field for the other value of ϵ which are in the range $\frac{1}{20} \leq \epsilon \ < 1$.

A. Generalized Emden-Fowler equation

Consider the following general differential equation

$$\ddot{\overrightarrow{A_k}} + F\dot{\overrightarrow{A_k}} + \frac{k^2}{a^2\gamma^2}\overrightarrow{A_k} = 0\,. \tag{A.1}$$

Now, using the change of variables as $\overrightarrow{A}_k = b^{\frac{-1}{2}}\tilde{\overrightarrow{A_k}}$, and define conformal time as $\eta = \int a^{-1}dt$ then

$$b^{\frac{-1}{2}}a^{-2}\tilde{\overrightarrow{A}}'' + b^{\frac{-1}{2}}a^{-2}\tilde{\overrightarrow{A}}'\left[(F-H)a - \frac{b'}{b}\right] +$$
$$\tilde{\overrightarrow{A}}\left[-\frac{F}{2}a^{-1}b^{\frac{-3}{2}}b' + b^{\frac{-3}{2}}a^{-2}\left(\frac{H}{2}ab' - \frac{b''}{2}\right) + \frac{3}{4}a^{-2}b^{\frac{-5}{2}}b'^2 - \frac{k^2}{a^2(\gamma^2)}\right] = 0\,. \qquad \text{(A.2)}$$

The second term can be cancelled provided that

$$(F-H)a = \frac{b'}{b} = a\frac{\dot{b}}{b}\,. \qquad \text{(A.3)}$$

and finally one obtains

$$\tilde{\overrightarrow{A}}'' + \tilde{\overrightarrow{A}}\left(\left[-\frac{a^2}{4}(F^2-H^2) - \frac{a^2}{2}(\dot{F}-\dot{H})\right] + k^2(\gamma^2)^{-1}\right) = 0\,. \qquad \text{(A.4)}$$

So, for the case $\gamma(\phi) = \frac{1}{\sqrt{1-f(\phi)\dot{\phi}^2}}$ one finds

$$\tilde{\overrightarrow{A}}'' + [k^2 - f(\eta)]\tilde{\overrightarrow{A}} = 0\,, \qquad \text{(A.5)}$$

where

$$f(\eta) = k^2 f\dot{\phi}^2 + \frac{a^2}{4}\left[(F^2-H^2) + 2(\dot{F}-\dot{H})\right]. \qquad \text{(A.6)}$$

This equation was called "Generalized Emden-Fowler" equation.

Acknowledgments

I would like to thank Dr. M. Alishahiha for valuable discussion. I like to thank M.R. Garousi, A. Ghodsi, A. Mosaafa, S. Rahvar, S. Sheikh-Jabbari, E. Silvrstein, A. Tavanfar, M. Torabian, Torkiha and D. Tong for comments. I would especially like to thank S. Movahed for valuable helps in numerical analysis of the problem.

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